REDUCED SEARCH SPACE FOR THE OPTIMAL DESIGN OF LINEAR PHASE FIR FILTER

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ABSTRACT

Branch and Bound Technique (BBT) is an optimal method for digital filter design in the discrete space. However, this algorithm is increasingly complex, as it requires an exhaustive search of the optimal solution in the discrete space. BBT may only be used for filter length $N \leq 8$.

The aim of this paper is to improve the convergence of the BBT algorithm. Therefore, we propose a novel method, which efficiently localizes the reduced region that contains the optimal solution. The proposed method tends to reduce the algorithmic complexity of BBT, preserving the optimality of the method. In this paper, we also schow, the performance of the method on few examples.

I. INTRODUCTION

The problem of digital FIR filter design has been solved since the development of Parks-MacClellan (PMC) algorithm [2]. However, the implementation of the obtained coefficients yields another problem that is discrete-coefficient FIR digital filter design. This new problem results from the constraint that the coefficients can only have discrete values in a processor. To solve this problem, several algorithms have been proposed. However, These algorithms are almost Ad-hoc and were not able to ensure the optimality of the solution for the discrete space.

One of the most used techniques for solving the problem of the optimality of the solution is Branch and Bound Technique (BBT) [3, 4]. In [4], BBT algorithm makes use of an exhaustive search to find for the optimal solution based on implicit enumeration techniques in the discrete space E^N. This algorithm (BBT) has the advantage of ensuring the optimality of the solution, however, it requires an expensive computing cost. Although the different attempts to improve the algorithm convergence, BBT still cannot be used for filter length $N>8$.

To solve this problem, many optimisation methods have been applied to discrete coefficients FIR digital filters design like Depth First Search [5], Simulated Annealing (SA) [6], Genetic Algorithm (GA) [7] and Tabu Search (TS) [8]. These techniques have proven to be effective in many cases and allow designing higher order filters, but do not guarantee the optimal solution.

The objective of this work is to propose an approach that allows an optimal digital filter from discrete coefficients and with lower algorithmic complexity than that of BBT. The aim is to reduce the search domain of the discrete coefficients without losing the optimality of BBT. We are concerned with the limitation and the localisation of the discrete solution space E^N_r with $E^N_r \subset E^N$.

In Section II, the formal definition of filter design in discrete space is given. The proposed approach to reduce the solution space is formulated in Section II and described in Section IV. Experimental results are given in Section V.

II. PROBLEM STATEMENT

The design of digital filter with length N and discrete coefficients consists of an exhaustive search in a discrete space $E_{proc}{}^N$, for a solution composed of N coefficients $[h(1), \ldots, h(N)] \in E_{proc}{}^N$, that presents the minimal error, according to the criteria: Least Squares Error, minmax.

In this paper, we are concerned with linear phase FIR digital filter. Four possible cases [1] rise for this kind of filters respective to N (even, odd) and h(n) (symmetric or asymmetric). We only consider the case of N even and h(n) symmetric in this paper. Extension to the other cases is straightforward. The considered case follows this form:

$$H(\omega) = e^{-j\omega(N-1)/2} A(\omega) \tag{1}$$

A (ω) is the frequency response amplitude that is written like this:

$$A(\omega) = \sum_{n=1}^{N/2} 2h(n)\cos(\omega(\frac{N+1}{2} - n)) \tag{2}$$

We notice that the number of linear phase FIR filter coefficients to calculate is equal to N/2. Therefore, the solution space is reduced to $E_{proc}{}^{N/2}$.

To improve the convergence speed of the algorithm BBT, we propose to reduce the search space to $E_{proc}{}^{N/2}$. Therefore, a set of conditions is exploited from the filter gabarit.

III. CONDITIONS FROM THE FILTER GABARIT AND INTERPRETATION

The filter specifications are generally driven by its gabarit. The amplitude A(ω) of the frequency response of the desired filter should range in $[1+\delta_p \;\; 1-\delta_p]$ for the Pass Band (PB) and in $[\delta_a$ and $-\delta_a]$ for the Stop Band (SB). δ_p and δ_a are the tolerances in PB and SB.

As an example, the lowpass filter case can be formulated as follows:

$$1-\delta_p \leq A(\omega) \leq 1+\delta_p \quad \forall\, \omega \in [0\,,\, \omega_p] \tag{3}$$

and $$-\delta_a \leq A(\omega) \leq \delta_a \quad \forall\, \omega \in [\omega_a\,,\, \pi] \tag{4}$$

ω_p and ω_a are respectively the pulses at PB end and SB begin.

Using (3) and (4), we have the conditions for the discrete pulses ω_i, which are used afterwards for the limitation of the search space. Actually, there is no condition on the number of pulses, just to find the smallest search space we have to choose the highest possible number of pulses. In the other hand, an increasing number would increase the search time, which is necessary to find the reduced solutions space. A trade-off has to be, however, found. During our experiments, we fixed the number of pulses to 4xN with a uniform distribution in the interval $[0\ \omega_p]\cup[\omega_a\ \pi]$.

$$\omega_{i=1\ldots m} \in [0\ ,\ \omega_p] \text{ and } \omega_{i=m+1\ldots 4N} \in [\omega_a\ ,\ 0.5]$$

To interpret these conditions, it is useful to look to this example (N=4).

Example: Let us consider a lowpass filter design with symmetric coefficients where the:

Pulses: ω_p=0.318 and ω_a=0.371

Tolerance: $\delta_p=\delta_a=\delta=0.47$

and L_{proc}=5 bits in fixed point, (sign bit included).

In this case, the solution space $E_{proc}^{4/2}$ has two dimensions (surface) in which the discrete solutions are represented by a grid of 1024 sparse points solutions (figure 1), according to the desired number of bits, with a quantization pace of q=*0.0625*.

Using the PB conditions, the correspondent Equation (3) is decomposed as follows:

$$2\,h(1)\cos({}^{3\omega_i}\!/_2) + 2\,h(2)\cos({}^{\omega_i}\!/_2) \le 1+\delta$$

and

$$2\,h(1)\cos({}^{3\omega_i}\!/_2) + 2\,h(2)\cos({}^{\omega_i}\!/_2) \ge 1-\delta$$

These two equations, which depend on the pulse ω_i, can be interpreted as two constraints (upper and lower). In the example above, these constraints define a space between parallel lines D_{is} and D_{ii} respective to each pulse $\omega_i\in[0, \omega_p]$.

The upper and lower constraints, for ω_1=0, are represented in Figure 1 using respectively the lines D_{1s} and D_{1i}. The candidate solutions are between the lines D_{1s} and D_{1i} where the optimal solution should be included.

We notice, then, the advantage of this approach that discards a huge number of non-candidate-solutions at once by simply using the equations defining the filter gabarit.

IV. SEARCH SPACE LIMITATION

A simple approach is used to limit the search space. It is subdivided into two steps. The first step consists of defining an initial closed space in the coefficients space. The successive reduction of the initial space is performed in the second step until reaching a compact space.

IV.1. Searching the Initial Closed Space

In the previous example (Section III), we showed that we could formulate two constraints, for each pulse, and represent them using two boundaries in the discrete space. To form a closed space, we have to consider two pulses, and the intersection between two pairs of parallel lines will define the closed space. These pulses have to be accurately chosen as they drive the convergence of the method. Actually, there is no method that allows determining the best pulses. In the case of the previous example (Section III), we chose ω_1=0 in PB and $\omega_{m+1}=\omega_a$ in SB. Equations for these pairs of lines that are depicted in Figure 1 and corresponding to the two frequencies, are given below:

$$2\,h(1) + 2\,h(2) = 1+\delta \tag{4}$$

$$2\,h(1) + 2\,h(2) = 1-\delta \tag{5}$$

$$2\,h(1)\cos({}^{3\omega_a}\!/_2) + 2\,h(2)\cos({}^{\omega_a}\!/_2) = \delta \tag{6}$$

$$2\,h(1)\cos({}^{3\omega_a}\!/_2) + 2\,h(2)\cos({}^{\omega_a}\!/_2) = -\delta \tag{7}$$

The initial space obtained is localized using the peaks A, B, C, and D shown in Figure 1. These peaks are obtained through the intersections between D_{1s}, D_{1i}, D_{m+1s}, D_{m+1i}.

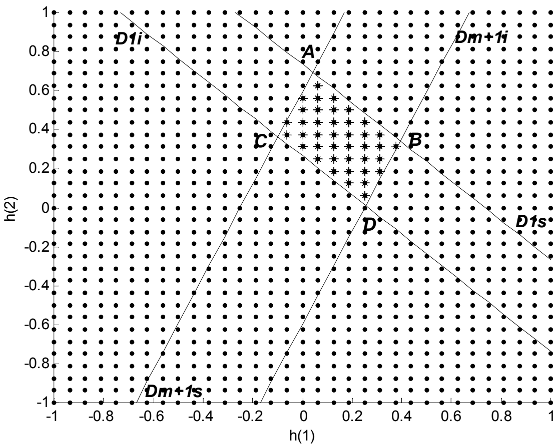

Figure 1. Interpretations in the Constraints Space $\{1-\delta_p \le A(0) \le 1+\delta_p\}$ and $\{-\delta_a \le A(\omega_a) \le \delta_a\}$.

The intersection between two pairs of lines yields a closed space ABCD

The parallel lines (D_{1s}) from Equation (4) and (D_{1i}) from Equation (5) cannot be crossed. As a consequence, these lines cannot result into a peak. However, they cross the lines (D_{m+1s}) and (D_{m+1i}) from Equations (6) and (7) respectively in A and B and in C and D. Based on this observation, the only lines pairs to consider for computing the peaks (A,B,C and D) coordinates that yield intersection points are the line pairs from Equations (4,6), (4,7), (5,6) and (5,7). The equations set derived is:

$$2.[\mathbf{A}].[\mathbf{h}]_{A..D} = [\mathbf{B}] \tag{8}$$

where

$$[\mathbf{A}]=\begin{bmatrix} 1 & 1 \\ \cos(\frac{3\omega_a}{2}) & \cos(\frac{\omega_a}{2}) \end{bmatrix}$$

$$[\mathbf{h}]_{A..D}=\begin{bmatrix} h(1)_A & h(1)_B & h(1)_C & h(1)_D \\ h(2)_A & h(2)_B & h(2)_C & h(2)_D \end{bmatrix}$$

is the matrix of the peaks coordinates A, B, C and D.

And

$$[\mathbf{B}]=\begin{bmatrix} 1+\delta & 1+\delta & 1-\delta & 1-\delta \\ \delta & -\delta & \delta & -\delta \end{bmatrix}$$

and finally, the peaks coordinates A, B, C and D are simply obtained by solving Equation (8).

$$[\mathbf{h}]_{A..D} = 1/2.[\mathbf{A}]^{-1}.[\mathbf{B}]$$

The inversion of the matrix [**A**], for solving Equation (8), is straightforward. We can show that this is invertible as all matrix elements are independent.

From this first step, we notice that the constraints resulted from the filter gabarit, helped to limit the search space to 40 points solutions candidates. This is a reduced number compared to the initial number of solution (1024). This number is fixed by the dimensions of the initial space ABCD, which not only depend on the pulses chosen but also depend on the desired tolerance δ. The choice of the pulses influences the convergence of the algorithm while δ has more influences on the final number of the solutions.

The solutions limited by the space ABCD are temporary considered as candidates, as we only can show that they verify the chosen constraints at $\omega_1=0$, and $\omega_{m+1}=\omega_a$

Outside these frequencies, the amplitude of the solutions is definitely different. Thus, there exist solutions not satisfying other chosen constraints at different pulses than ω_1 and ω_{m+1}. The selection and removal of these solutions will further reduce the initial search space. To more reduce the search space, we will consider other constraints for pulses in the interval $]0,\omega_p]U]\omega_a,\pi]$.

IV.2. Reduction of the Search Space

Within this step, the upper and lower constraints are separately considered. An iterative technique is used to reduce the initial search space and applied for both constraints.

As shown in the previous section, a boundary is associated to each constraint. Thus, depending on the position of a new line and the resulted space in the previous step (We primarily consider the initial space), we have 3 cases.

Case 1: the new line crosses the space. This is an interesting constraint as it reduces further the search space. The new line D_{ki} in Figure 2 crosses the region ABCD passing by the points E and F and divides it into two parties.

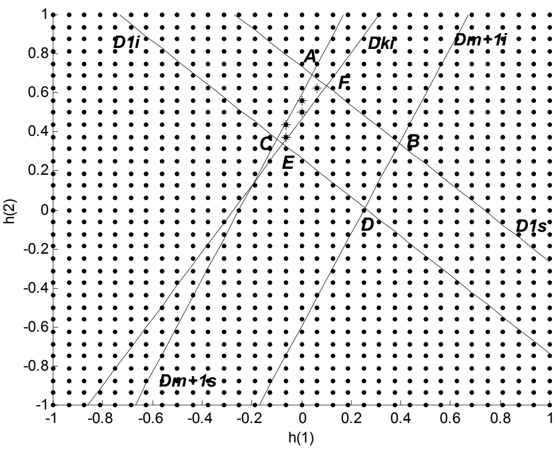

Figure 2. Reduction of the Search Space.
Case 1: The Line Crossing the Space

The new space is bounded by the peaks A, C, E, and F, and contains 5 candidate solutions. In this case, the region BDEF is discarded from the search because all solutions belonging to it don't verify the new constraint. Therefore, B and D are substituted by E and F for localizing the new search space.

Case 2: The new line does not cross the search space and this latter is contained in the admissible region respective to the line (example of the line D_{ms} in Figure 3). All solutions bounded by the space are still candidate. The new line does not result in a reduction of the search space, which is not interesting for our purpose.

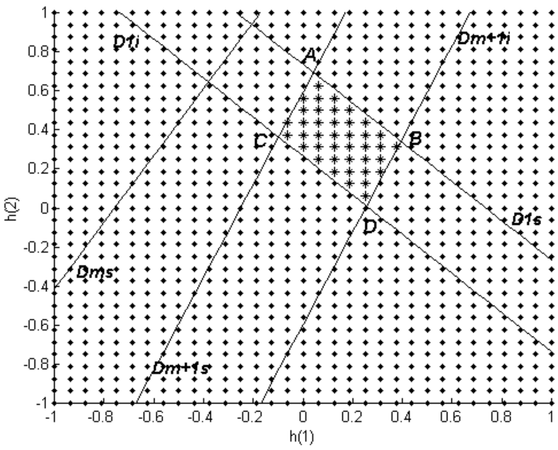

Figure 3. Reduction of the Search Space.
Case 2: The Line not Crossing the Space

Case 3: The new line does not cross the search space and this latter is contained in the non-admissible region respective to the line. In this case, all solutions located in the space ABCD do not verify this new constraint and, thus, they are not candidate. An empty solution space will result and the problem cannot be solved. This is a case for $\delta<\delta_{opt}$, where δ_{opt} is the optimal tolerance in the continuous space. Only case 1 allows the reduction of the search space, and is, therefore, interesting for identifying the appropriate constraints. The space reduction procedure is performed for all 2x4xN constraints.

V. EXPERIMENTATIONS

V.1. Example with N=4

The example presented in Section III is tested in this subsection in order to illustrate the minimal space resulting. We added to Figure 1 the boundaries corresponding to all constraints formulated starting from the following frequency:

PB: [0 0.0318 0.0636 0.0954 0.1272 0.1590 0.1908 0.2226 0.2544 0.2862 0.3180].

SB: [0.3710 0.4033 0.4355 0.4678 0.5].

The lines corresponding to the PB frequencies are shown in solid lines while dashed lines depict boundaries for SB frequencies (Figure 4).

The targeted reduced space is the solutions space that fulfil all conditions, depicted in bold in Figure 4. Each solution belonging to this space is a candidate, that is also verified by means of the continuous solution using the PMC algorithm shown with ''+'' in Figure 4. The number of candidate solutions kept in the resulting space is equal to 5. Compared to the total number of possible solutions in the initial space E_{proc}^2, the reduction is drastic. As a consequence, the complexity – also the computing time – required by (BBT) is significantly reduced.

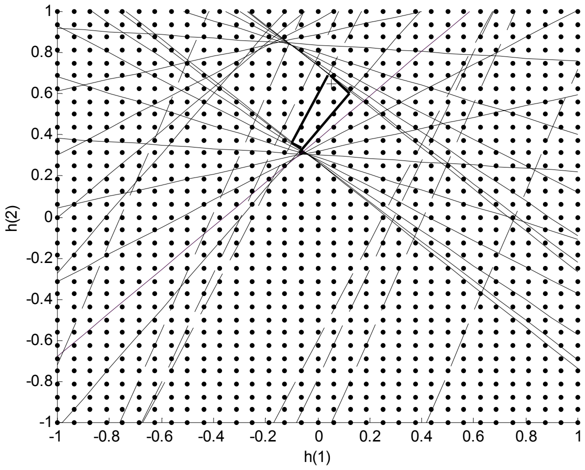

Figure 4. Localization of the Reduced Space Containing the Candidate Solutions.

V.2. Comparison between the Numbers of Solutions

In this subsection, a comparison between the number of solution nb_T using BBT and that nb_r using our approach is given for filter lengths N=14 … 22 and L_{proc}=12 bits (Fixed point). The obtained results are given in Table 1.

N	nb_T	nb_r	nb_r/ nb_T
14	1.9343e+025	4096	2.1176e-022
16	7.9228e+028	34560	4.3621e-025
18	3.2452e+032	2222640	6.8490e-027
20	1.3292e+036	327680	2.4652e-031
22	5.4445e+039	5292000	9.7199e-034

Table 1. Comparison between BBT and Our Approach in term of Complexity.

A drastic reduction of nb_r compared to nb_T is shown in Table 1. The numbers of nb_r depicted in Table 1 are not minimal. This is noticed from the increasing number of nb_r in the cases of N=18 and N=22. These numbers (nb_r) mainly depend on the tolerance δ that was arbitrarily chosen in our method.

It is clear that the computing time is proportional to the number of solutions to test in the search space, which shows the importance of this work as the solutions space has been reduced and thus, the convergence of BBT is accelerated for the filter design. We also notice that the reduction rate $nb_{r/}$ nb_T est very low for high filter length N, therefore, this technique is efficient for high filter order than low filter order.

In case of N=22, The minimal values h_{min} and maximal values h_{max} of the coefficients, which are the projection of the reduced space, obtained by this approach are listed below:

h_{min}	h_{max}	-0.031750821	-0.03029524
-0.00709509	-0.00690810	-0.02102270	-0.01948206
0.03922196	0.03966655	0.08179348	0.08315493
-0.00891269	-0.00814312	-0.09560910	-0.09412101
-0.01670367	-0.01565370	-0.02308614	-0.02142280
0.03935306	0.04064022	0.56062739	0.56238365

The design of a filter with identical specifications, using PMC algorithm [2] provides the following coefficients:

h(1) = -0.00692909 = h(22) h(7) = -0.02009873 = h(16)
h(2) = 0.03965419 = h(21) h(8) = 0.08270339 = h(15)
h(3) = -0.00821405 = h(20) h(9) = -0.09469138 = h(14)
h(4) = -0.01586129 = h(19) h(10)= -0.02213961= h(13)
h(5) = 0.04025149 = h(18) h(11)= 0.56157526 = h(12)
h(6) = -0.03082413 = h(17)

We can easily verify using these results that $h \in [h_{min}\ h_{max}]$. This would confirm, as before, that the resulting reduced space would contain the optimal solution.

VI. CONCLUSIONS.

In this paper, we are concerned with a development of an efficient algorithm for the limitation of the search space required by the Branch and Bound Technique (BBT). The results of our approach is very useful as BBT using the exhaustive search is limited to filter length N≤8 because of the computing cost. The limitation of the search space using conditions on the filter gabarit allows accelerating the convergence of the filter design and also the design of higher filter orders. As an example, we have shown that our approach reduces the BBT complexity by a factor of 10^{33} for filter length N=22 with L_{proc}=12 bits.

Although, the results shown in this paper are acceptable, they still are not optimal. Indeed, the dimensions of the reduced space depend on δ and therefore, a further investigation to improve the obtained results is in our perspective.

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