A NEW APPROCH TO DIGITAL FIR FILTER DESIGN USING THE TABU SEARCH

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ABSTRACT

The Tabu Search method has proved to be efficient in many cases applied to digital filter design. However, the starting point chosen can affect the output quality, the computing time and the performance. In this paper, we present a new strategy to improve the output quality of the Tabu Search algorithm using as starting point, that obtained with the Sequential and the Progressive Search method. We will also show that the choice of the fixed-point binary representation provides better results than those of the power of two.

Keywords : *Tabu Search ´TS´, Depth First Search 'DFS', Sequential and Progressive Search 'SPS', Simulated Annealing 'SA'.*

1. INTRODUCTION

The Tabu Search (TS) tool [1],[2] has proved to be both versatile and easy to use, thus rapidly allowing its customisation to different optimisation applications. It exploits some of the most effective search techniques taken from the literature, as well as some new search strategies. In the case of digital FIR filter design where the coefficients are represented by a finite number of bits, TS algorithm has also provided better results than those of the literature such as ´Simulated Annealing` (SA)[13]. SA finds its best application in the design of special filters, such as Nyquist and cascade form FIR filters, where we may have numerous or conflicting constraints. Therefore, the computing cost is expensive.

The Sequential and Progressive Search (SPS)[3] method based on an extrapolation of the final solution from a starting one is also prohibitive for high filter order. The starting solution is found using the Depth First Search (DFS)[12] in a 3 bits word length.

The performance of TS can be affected by the choice of the starting solution [1][2][11]. Although the results obtained with TS are better than those of the literature, we will show that we can further improve them using a studied starting point such as SPS starting point. We will also show that we can also improve these results using an appropriate binary representation, i.e., fixed point representation instead of power of two.

This paper is structured as follows. In section 2, we present the problem statements and the characteristics of the error criterion chosen. In section 3, the descriptions of the Tabu Search method (TS), the sequential and progressive search method (SPS) and the binary representation are given. The results reported on section 4 deal with conventional minmax optimisation of FIR digital filter and are compared to those of other methods.

2. PROBLEM STATEMENTS

Let us consider the design of N-1 order linear phase FIR digital filter with a frequency response H(f), usually written as

$$H(f) = \sum_{k=0}^{N-1} h_k e^{-j2\pi fk} \qquad (1)$$

There exists four cases of linear phase FIR filter depending on the filter order, even or odd, and on the kind of symmetry of its impulse response `$h_k$`, positive or negative. In [5], it was shown that the frequency response amplitude of the four cases of linear phase filters can be written in the form

$$P_n(f) = \sum_{k=0}^{n-1} a_k cos 2\pi fk \qquad (2)$$

where the number of terms, n, is:

$$n = N/2 \text{ or } (N-1)/2 \text{ or } (N+1)/2$$

and a_k, related to h_k, is the resulting shifted sequence depending on the considered case.

The function $P_n(f)$ is compared with a desired frequency response amplitude $D(f)$ using a minmax criterion, as done in the usual optimal linear phase FIR filter design with infinite precision [6]. During the optimisation, the objective function to minimise $f(a)$ is

$$f(a,G) = \max_{f \in F} W(f) \left| D(f) - G^{-1} P_n(f) \right| \qquad (3)$$

- F : the disjoint union of all the frequency bands of interest.
- G: filter gain
- $W(f)$: a weighting function defined on F.
- $D(f)$: the desired frequency response amplitude.

Using Eq. (2) in Eq. (3) gives

$$f = \max_{f \in F} W(f) \left| D(f) - G^{-1} \sum_{k=0}^{n-1} \mathrm{a_k} \cos 2\pi \mathrm{fk} \right| \qquad (4)$$

The filter coefficients are restricted to the discrete values allowed by 'b' bit binary word length.

3. METHODS

3.1.Tabu Search method (TS)

Tabu Search [7]-[10] is a metaheuristic method that leads the search for the good solution on the minmax sense making use of flexible memory systems which exploit the history of the search. TS consists on the systematic prohibition of some solutions to prevent cycling and to avoid the risk of being trapped in local minima. New solutions are searched in the neighbourhood of the current one. The neighbourhood is defined as the set of the points reachable with a suitable sequence of local perturbations, starting from the current solution.

One of the most important features of TS is that a new configuration may be accepted even if the value of the objective function f(a) is greater than that of the current solution. In this way it is possible to avoid being trapped in local minima.

Among all the visited solutions the best one is chosen. This strategy can lead to cycling on previously visited solutions. To prevent this effect, the algorithm marks as "tabu" certain moves for a number of iterations. To do this, a so-called tabu list T of length TT=|T|, named *tabu-tenure*, which can be fixed or variable, is introduced.

Some aspiration criteria which allow overriding of tabu status can be introduced if that move is still found to lead to a better cost with respect to the cost of the current optimum.

This is a characteristic aspect of TS methods, whose main novelty is the use of flexible memory systems for taking advantage of the history of the search.

The previously described memory is the so-called 'short term memory'. A second kind of memory called 'long term' can also be implemented.

Two main important long term memory concepts, which should be evaluated, are intensification and diversification strategies. Intensification strategies are based on the idea of encouraging move combinations and solution features historically found to be good. Diversification strategies, on the other hand, are designed to drive the search into new promising regions.

Summing up, the performance of a TS algorithm depends on the proper choice of the neighbourhood of a solution, on the number of iterations for which a move is kept as tabu, on the aspiration criteria, on the best combination of short and long term memory and on the best balances of intensification and diversification strategies.

These choices are closely linked to the problem at hand and often require expensive "trial and error" processes.

In [1],[2] the TS method uses as starting point Parks- Mc Clellan coefficients or random coefficients. In these cases the results quality, the performance and the computing time, depend on these starting points. Our aim in this paper is to provide a suitable starting point. Our starting point is that of the SPS method described below.

3.2. Sequential and Progressive Search method (SPS)

The SPS method is based on the DFS method. Its major issue of concern is the determination of a good branching strategy. This strategy is detected after a study about the effect of the quantization error on the frequency response, and an examination of the DFS characteristics.

The DFS cannot be applied for a large processor word length 'b', due to the high number of discrete admissible values. Therefore, we use this technique to found an optimal solution in a lower word length 'lb' (lb < b). After, we perform an extrapolation to reach a final solution on 'b' bits wordlength under a set of constraints.

The SPS begins from the discrete optimal starting solution found by the DFS algorithm in a lower wordength, 3 bits in our case. Also, we increase the precision of the coefficients after defining the search interval for each one in the upper wordlength.

In the following sections, the starting solution of the SPS algorithms is chosen as TS starting solution.

3.3. The binary representation

In this paper, we apply two binary representation : the power of two and the fixed point representation. The power of two is used in order to compare the TS results to those of the literature such as SA, and TS with Parks-Mc Clellan starting solution. The fixed-point representation is used in order to provide an improvement of the TS results quality.

The coefficient space in both representations is defined in a ´b´ bits wordlength as follow:

- Power of two:

$$D = \left\{ \begin{array}{l} a : a = \sum_{k=1}^{2} c_k \cdot 2^{-g_k}, \quad c_k \in \{-1,0,1\}, \\ g_k \in \left\{1,2,\ldots,2^{\frac{b}{2}-2}\right\} \end{array} \right\}$$

- Fixed point:

$$D = \left\{ \begin{array}{l} a : a = s.\sum_{k=1}^{b-1} c_k \cdot 2^{-k}, \quad c_k \in \{0,1\}, \\ s = \{-1,1\} \end{array} \right\}$$

4. RESULTS

The TS algorithm using different starting point was developed in ANSI C and tested on 400 MHz Pentium machine. The results obtained are presented and compared to algorithms in [13] and in [2]. A filter with length 21 and 6 bits processor wordlength, excluded the sign bit is denoted by '21/6'. Firstly, we will compare the TS performance with that of SA reported in [13]. To better compare the two algorithms, in Table 1 we present both the maximum weighted error given by (3) and the normalised peak weighted ripple δ/B, where δ is the peak weighted ripple and B is the mean value of the passband gain.

In this table the results of five low-pass filters design are reported, with a passband cut-off normalised frequency of 0.15 and a stopband edge of 0.25. The number of sampling frequencies in (3) is 512, gain G varies in the range 0.5-1.0,

The overall quality of the filters designed with TS and SA is almost the same (up to 0.1 dB) in all cases. However, the two algorithms are remarkable different in terms of computational costs expressed by the total number of function evaluations. In fact, TS requires from 10% to 50% less calculation than SA method.

Filter length	Starting point		Simulated Annealing (optimum point)		Universal Tabu Search (optimum point)		
	Maximum error value	δ/B [dB]	δ/B [dB]	Number of function evaluations	Maximum error value	δ/B [dB]	Number of function evaluations
27/9	0.0126	-33.5	-41.3	849000	0.008498	-41.4	710000
29/9	0.0153	-33.5	-43.1	939000	0.007016	-43.1	630000
31/9	0.0153	-33.5	-43.1	1060000	0.007016	-43.1	627000
33/9	0.0129	-35.7	-44.7	1026000	0.005797	-44.7	725000
35/9	0.0118	-33.5	-44.7	1105000	0.005797	-44.7	546000

Table 1. Results of 5 filter designs and comparison with simulated annealing. Normalised cut-off frequencies are 0.15 and 0.25.

N/b	Infinite precision	Rounded (Power of two)	TS with Parks-Mc Clellan starting solution		TS with random starting solution		TS with SPS starting solution	
			f(a)	Time s	f(a)	Time s	f(a)	Time s
8/15	0.057863	0.125033	0.099016	2.03	0.1536214	32.05	0.0979985	2.03
21/6	0.001989	0.046875	0.0157447	10.32	0.0658742	257.56	0.0153654	6.24
8/7	0.057863	0.125663	0.099016	1.04	0.112546	38.54	0.0987965	0.66
20/7	0.003429	0.0621487	0.0120481	10.17	0.0126542	336.5	0.0120481	2.65
16/19	0.136302	0.167227	0.140713	6.53	0.152633	59.50	0.1365435	4.56

Table 2. Results of TS algorithm with different starting solutions compared to reference method in power of two

N/b	Infinite precision	Rounded (Power of two)	TS with Parks-Mc Clellan starting solution		TS with random starting solution		TS with SPS starting solution	
			f(a)	Time s	f(a)	Time s	f(a)	Time s
8/15	0.057863	0.057856	0.057681	1.27	0.059980	36.68	0.05709	1.22
21/6	0.001989	0.051301	0.031250	5.88	0.037500	289	0.03125	3.25
8/7	0.057863	0.063677	0.0636766	1.32	0.0758421	85.56	0.06355	1.01
20/7	0.003429	0.021929	0.015625	5.49	0.018750	268.65	0.015625	3.54
16/19	0.136302	0.136303	0.135806	5	0.145610	78.65	0.13569	4.1

Table 3. Results of the Tabu Search algorithm with different starting solutions in the fixed point representation

Secondly, it is shown how a proper choice of the starting solution can greatly affect the goodness of the solutions. The three first filters in the Table 2 and 3 have the passband edges (0,0.159) and the stopband edges (0.295,0.5), the fourth filter has the passband edges (0,0.307) and the stopband edges (0.35,0.5) and the last filter has the passband edges (0,0.08) and the stopband edges (0.16,0.5). All the filters have equal weights in passbands and stopbands and unitary gain. In all examples the results of TS with SPS starting solution are better than others in the same case. As seen in Table 2, the computing time for TS using a random starting solution is the biggest for a less performance. However, the results of TS using Parks- Mc Clellan solution requires a quasi-equal time to that of TS using SPS starting solution for a less performance. In table 3, we improve the performance of TS algorithm using a more representative binary representation fixed point‘. We can see that the results obtained are better in most cases to those of table 2. The reduction of computing time and improvement of performance are due to the high number of admissible values in the fixed-point representation. This provides a fast and a good solution.

5. CONCLUSION

The Tabu Search algorithm combines the most interesting and effective search techniques designed by several authors with new ideas and strategies aiming to satisfy simplicity and versatility. In this paper, an improvement of TS algorithm for digital FIR filter design using a new starting solution and a new binary representation is presented. The main target was to choose an independent starting solution from that of Parks-Mc Clellan one, which uses rounding to infinite precision solution. Therefore, we use a discrete starting solution directly chosen from the processor discrete space. The obtained results when compared to those of other strategies are better in all cases.
As a second improvement is the use of fixed-point representation which provides a larger coefficients space than that of power of two for the same wordlength. This yields better performance.
As a future work, the application of the Tabu Search algorithm for the IIR filter will be studied.

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