A SEQUENTIAL ROBUST METHOD TO FINITE WORDLENGTH COEFFICIENT FIR DIGITAL FILTER DESIGN

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ABSTRACT

This paper describes a novel branching strategy using the branch and bound technique for the design of finite word length optimal digital filters. Using an extrapolated solution, the presented technique offers good performances in a low algorithmic complexity for large processor wordlength. It also provides a large flexibility to different error criterion. The details of the algorithm and many examples are given and compared to the other methods.

I. INTRODUCTION

One of the most known optimisation methods in the coefficients discrete space is the Branch and Bound Technique (BBT). This was used to solve the discrete optimum problem. This method based on implicit enumeration techniques also requires an expensive computing cost [7,8,10,15,16]. Many exhaustive approaches are performed on the (BBT) basis. The Depth First Search (DFS) and Breath First Search (BFS) are the famous approaches. Their major issue of concern is the strategy of branching. (DFS) approach performs its search in the whole available discrete space. Therefore, the obtained solution is optimal. However, before optimisation, (BFS) performs an estimation of the discrete space susceptible to contain the optimal solution. Hence, the performance are not guaranteed. Furthermore, the computing cost is prohibitive in a large processor wordlength.

The rounding of infinite precision coefficients for linear phase FIR digital filters design [6] is the most widely used technique when low algorithmic complexity is required. However, the frequency response amplitude of the obtained filter do not usually fulfill the passband and/or the stopband frequency tolerances.

For discrete optimisation, it is often desirable to use algorithms whose output quality can be adjusted depending on the availability of resources such as computing time and precision. To solve this problem, many optimisation methods has been applied to discrete coefficients FIR digital filters design. Simulated annealing technique [2,3,4] has proven to be effective in many cases, but requires a large number of function evaluations and does not guarantee the optimal solution. Furthermore, this method is based on heuristic assumptions, which may lead to bad performance.

The linear integer programming formulation [1],[7]-[9] was applied as a discrete optimisation method on the minmax criterion. Although, it is possible to obtain an optimum result, the computing time required even with the high-speed supercomputer of today, prohibits the application of these techniques for high order filters.

In this paper, we present a novel concept to design finite wordlength coefficients filter. The performed method named Sequential and Progressive Search (SPS) is based on a robust branching strategy. Its aim is to reduce the discrete space using an extrapolation method and perform the optimisation in the same time. Compared to the Depth First Search method (DFS), the number of function evaluations is smaller and it depends on the filter length without degrading the performance of the algorithm [14]. It will be shown (SPS) and (DFS) algorithmic complexity related to the processor wordlength.

In section II, we present the problem statements and the characteristics of the error criterion chosen. In section III, (SPS) method description is given. The algorithm modules are depicted in section IV. The results reported on section V deal with conventional minmax optimisation of FIR digital filters. A comparison of the algorithm performance with other reference algorithm is also given.

II. PROBLEM STATEMENTS

The frequency response of N-1 order linear phase FIR digital filter is usually written as

$$H(f) = \sum_{k=0}^{N-1} h_k e^{-j2\pi fk} \tag{1}$$

In [5], It was shown that the frequency response amplitude of the four cases of linear phase (FIR) filters could be written in the form

$$P_n(f) = \sum_{k=0}^{n-1} a_k \cos 2\pi fk \tag{2}$$

where the number of terms, n, is:

$$n(= N/2 \text{ or } (N-1)/2 \text{ or } (N+1)/2)$$

and a_k is the resulting shifted sequence depending on the considered case. The function $P_n(f)$ is compared with a desired frequency response amplitude D(f) using a minmax criterion. The weighted approximation error e_n is given by

$$e_n = \min_{(coeff.a)} \max_{f \in F} W(f)\left|D(f) - P_n(f)\right| \qquad (3)$$

- F : the disjoint union of all the frequency bands of interest.
- W(f): a weighting function defined on F.
- D(f) : the desired frequency response amplitude.

Using Eq. (2) in Eq. (3) gives

$$e_n = \min_{(coeff.a)} \max_{f \in F} W(f)\left|D(f) - \sum_{k=0}^{n-1} a_k \cos 2\pi fk\right| \qquad (4)$$

The filter coefficients are restricted to the discrete values allowed by (bwl) bit binary word length.

III. OPTIMIZATION METHOD 'SEQUENTIAL AND PROGRESSIVE SEARCH' (SPS)

Let us consider $\{a_k, k=0,1,...., N/2\}$ the filter discrete coefficients designed with an extra constraint imposing a limit on the processor wordlength.

Using the fixed point representation, we can express the discrete coefficient a_k as a linear combination:

$$|a_k| = \sum_{j=1}^{bwl-1} y_{j,k} 2^{-j} \quad k = 0, 1,...., N/2. \qquad (5)$$

Where (bwl) is the binary bit allowed for the filter discrete design and 'j' is the binary bit indication. $Y_{j,k}$ is a bivalent variable only fixed to only take the values '0' or '1'. Hence, the frequency response amplitude $P_n(f)$ could be expressed as

$$P_n(f) = \sum_{=0}^{-1} s\left(\sum_{j=1}^{bwl-1} y_{j,k} 2^{-j}\right) \cos 2\pi fk. \qquad (6)$$

where s is the sign of 'a_k' , s (= -1 or +1).

It is not possible to find the optimal filter coefficients at 'bwl' bits wordlength processor using the DFS method, owing to the long computing time required. But we can calculate these filter coefficients in a lower wordlength, (bbN) binary bit (bbN ≤ bwl) with such a method. a_k could be expressed as

$$|a_{k(opt)}| = \sum_{j=1}^{bbN-1} y_{j,k(opt)} 2^{-j} \quad k= 0, 1,...., N/2. \qquad (7)$$

$a_{k(opt)}$: the coefficients related to the optimal digital filter at (bbN) wordlength.

$Y_{j,k(opt)}$: 'j' binary bit value of the $a_{k(opt)}$ coefficient.

The proposed method, named Sequential and Progressive Search (SPS) takes this optimal solution as a starting point (starting solution) for its branching strategy in order to design filters with a higher wordlength discrete coefficients. Using the (DFS) optimal solution in the minmax sense in the bbN binary bits wordlength, we calculate with the SPS algorithm the solution at bwl bits (bbN ≤ bwl).

First, the coefficients are found at the (bbN+1) binary bits on the minimax sense using a local investigation in a reduced discrete search space. This reduced space is defined using the previous solution. Then we increase gradually the wordlength and calculate the new solution till reaching the (bwl) (the required word length). For each step 'i', we define a lower bounding function $e_{n\,i}$, which can be written as

$$e_{n\,i}(a) = \min_{(coeff.a)} \max_{f \in F} W(f)\left|D(f) - \sum_{k=0}^{n-1} a_k \cos 2\pi fk\right| \qquad (8)$$

$e_{n\,i}(a)$ is defined as the lowest value of the error function for a_k solving the following program :

$$a_k^{bbN+1} \le a_k^{bbN} + \mu \quad k=1,..., N/2 \qquad (9a)$$

$$a_k^{bbN+1} \ge a_k^{bbN} - \mu \quad k=1,..., N/2 \qquad (9b)$$

a_k^{bbN} : is the coefficients a_k represented by bbN bits

μ: interval chosen to contain the solution.

The implementation of a continuous value in two different wordlengths of the fixed-point representation does not offer two equal discrete values. The maximum difference between these discrete values is the quantization error '±LSB' (least sided bit) due to both truncation (±LSB) and rounding (±1/2.LSB). Hence, we have overestimate the μ value between the bbN and bwl wordlengths to

$$\mu = LSB = 2^{-(bbN-1)}. \qquad (10)$$

Substituting Eq. (10) and Eq. (7) in Eq. (9)

$$\sum_{j=1}^{bbN} y_{j,k} 2^{-j} \le \sum_{j=1}^{bbN-1} y_{j,k(opt)} 2^{-j} + 2^{-(bbN-1)} \qquad (11a)$$

$$\sum_{j=1}^{bbN} y_{j,k} 2^{-j} \ge \sum_{j=1}^{bbN-1} y_{j,k(opt)} 2^{-j} - 2^{-(bbN-1)} \qquad (11b)$$

Developing Eq. (11) we obtain

$$\sum_{j=1}^{bbN-2} y_{j,k(opt)} 2^{-j} + \sum_{j=bbN-1}^{bbN} y_{j,k} 2^{-j} \le \sum_{j=1}^{bbN-2} y_{j,k(opt)} 2^{-j} + y_{bbN-1,k(opt)} 2^{-bbN} + 2^{-(bbN-1)}. \qquad (12a)$$

$$\sum_{j=1}^{bbN-2} y_{j,k(opt)} 2^{-j} + \sum_{j=bbN-1}^{bbN} y_{j,k} 2^{-j} \ge \sum_{j=1}^{bbN-2} y_{j,k(opt)} 2^{-j} + y_{bbN-1,k(opt)} 2^{-bbN} - 2^{-(bbN-1)}. \qquad (12b)$$

After simplifications, we have

$$y_{bbN-1,k} \cdot 2^{-bbN+1} + y_{bbN,k} \cdot 2^{-bbN} \le y_{bbN-1,k(opt)} \cdot 2^{-bbN} + 2^{-(bbN-1)} \qquad (13a)$$

$$y_{bbN-1,k} \cdot 2^{-bbN+1} + y_{bbN,k} \cdot 2^{-bbN} \ge y_{bbN-1,k(opt)} \cdot 2^{-bbN} - 2^{-(bbN-1)} \qquad (13b)$$

Hence,

$$y_{bbN-1,k} + y_{bbN,k} 2^{-1} \le y_{bbN-1,k(opt)} + 1 \qquad (14a)$$

$$y_{bbN-1,k} + y_{bbN,k} 2^{-1} \ge y_{bbN-1,k(opt)} - 1 \qquad (14b)$$

The problem is restricted as resolving two equations with two variables (x1,x2) on the form of

$$a1x1 + a2\,x2 \le b1 \qquad (15a)$$

$$a1x1 + a2\,x2 \ge b2 \qquad (15b)$$

(a1, a2, b1, b2) are constants.

a1=1, a2= 2^{-1}, b1=y_p+1, b2=y_p-1,

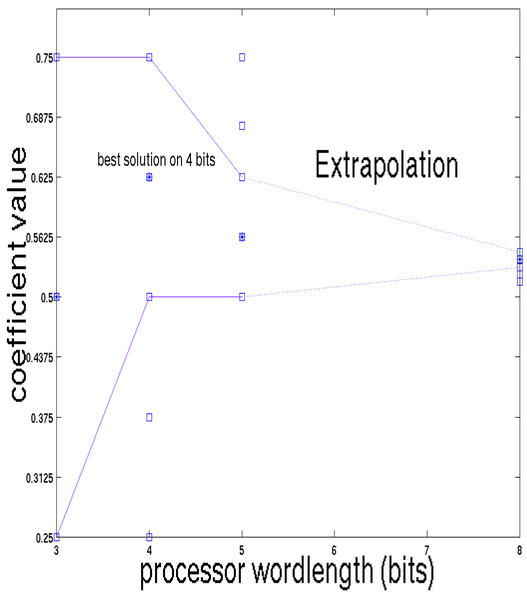

Figure1. SPS Procedure on one coefficient

and where y_p is the previous calculated optimal solution. (in this case $y_p = y_{bbN-1,k(opt)}$)
Hence, we obtain a small grid containing admissible values, from which we choose the solution sequence (x1, x2) which gives the smallest value of maximal weighted error. This procedure is iterated for each coefficient increasing the word length, until reaching the desired word length, in which the design of discrete coefficients digital FIR filter was required. Figure 1 represents on one coefficients the (SPS) procedure. The x-axis is and the y-axis are respectively the processor wordlength and coefficient value. The square is the admissible values in the reduced discrete space. The cross the coefficient after optimisation using (SPS).
In this paper, we have chosen the fixed point transformation as shown in Eq. 5 We can show that an extension to the other binary representation could be easily done.

IV. THE SEARCH STRATEGY

The SPS algorithm begins from the discrete optimal starting coefficients designed by (DFS) in a lower word length. Also, we determine the interval for each coefficient in the upper word length as in Figure 1.
Therefore, the formula Eq. (15) is resolved each wordlength incrementation using a local investigation at defined nodes (reduced discrete space).
The algorithm flowchart is described in Figure2. Its main characteristics, which are used for computational experiments, are described below.

- The branching strategy depends starting solution found by (DFS).
- The function evaluation is in the minmax sense. Using (DFS) algorithm, all the defined nodes are investigated.

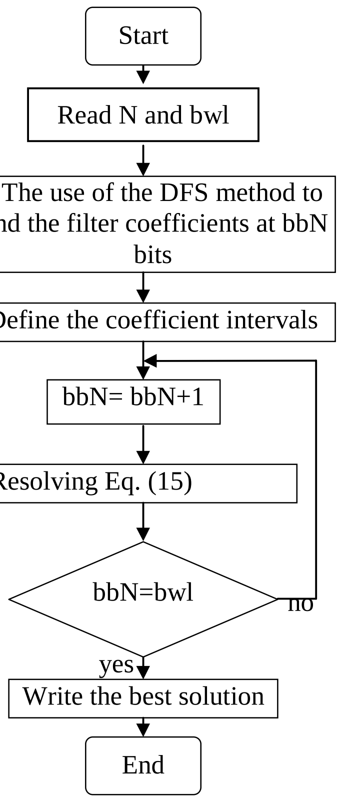

Figure2. The Strategy Search Flowchart

V.RESULTS

The (SPS) software algorithm was developed on MATLAB and tested on 300 MHz Pentium machine using cases reported in literature. The results obtained are presented and compared to those of algorithms in [4,15,16]. The reference numbers indicate where the filters are taken, (DFS), (BFS) and (SA). A filter with length 8 with 15 bits in quantization, excluded the sign bit is denoted by (8/15). The starting points are found by (DFS) in 3 bits processor wordlength. Filters in Table 2 have the frequency specifications represented in Table1. All filters have equal weight in passbands and stopbands. In Table 2 the results are given both in the weighted approximation error and the design times. All SPS performances are better than those obtained in the indicated references. Figure 3 represents the algorithmic complexity ratio between (DFS) and (SPS) algorithms according to the processor wordlength increase. It is shown the drastic time reduction using (SPS) when (bwl) increases.

N/bwl	Passband edges	Stopband edges
16/15	0 : 0.151	0.2050 : 0.5
8/15	0 : 0.411	0.4780 : 0.5
21/6	0 : 0.100	0.1125 : 0.5
12/19	0 : 0.178	0.2500 : 0.5
14/19	0 : 0.268	0.3750 : 0.5
16/19	0 : 0.358	0.4250 : 0.5

Table 1. Filters Frequency Specifications

N/bwl	N	bbN	bwl	Inf. Precision	Rounded	[ref]/ Time (sec)	SPS/ Time (sec)
16/15[15]	16	3	15	0.0871	0.0872	0.0762/ 650	0.0712/509
8/19[16]	8	3	19	0.0850	0.0999	0.0975/ 54	0. 0968/ 16
21/6[4]	21	3	6	0.0209	0.0722	0.0711/ 5	0.0468/ 79256
12/19[15]	12	3	19	0.0930	0.0930	0.0911/ 154	0.0895/ 112
14/19 [15]	14	3	19	0.0323	0.0323	0.0305/ 139	0.0294/132
16/19[15]	16	3	19	0.0766	0.0766	0.0747/ 1276	0.13580/10567

Table 2. Results & Comparison for Filter Design Cases Using the SPS Method.

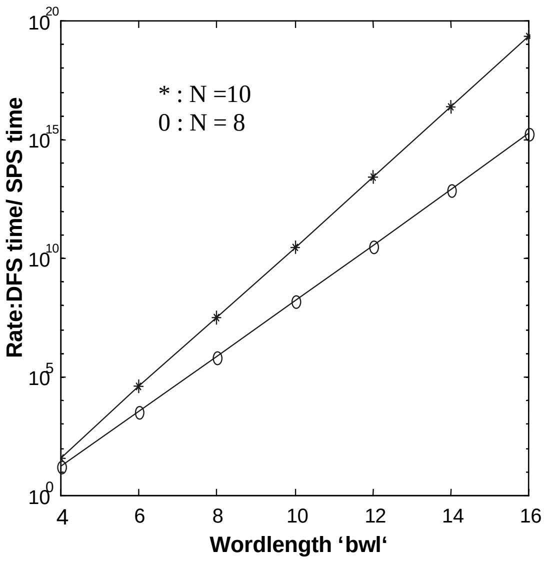

Figure3. Representation of the DFS time/SPS time according to different wordlengths for N:8 and 10

V. CONCLUSION

In this paper, the procedure of a new digital filter design method (SPS) in the discrete space is presented. An evaluation of algorithm performance compared to other algorithms (DFS), (BFS) and (SA) is given. The main feature of this approach is its applicability to the design of filter in a processor with a large wordlength. The computing time in such processor wordlength would be prohibitive using the Depth First Search (DFS). Therefore, it is shown the algorithmic complexity reduction using (SPS) algorithm when processor wordlength increases. In the examples, the limitation of the search domain does not seem to degrade the performance of the algorithm. The forthcoming target is the procedure extension to long filter order.

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