THE CONTOURLET TRANSFORM FOR IMAGE COMPRESSION

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ABSTRACT

The JPEG 2000 norm uses the bi-orthogonal wavelet transform for image decomposition. Wavelets are well adapted to point singularities (discontinuities), however they have a problem with orientation selectivity, and therefore, they do not represent two-dimensional singularities (e.g. smooth curves) effectively. This paper introduces and presents an evaluation of the contourlet transform for image compression, which has good approximation properties for smooth 2D functions. The contourlet finds a direct discrete-space construction and is therefore computationally efficient. The information gain resulting from this approach is demonstrated on several images with a minimal resolution of 1 Mega pixels. A comparative study is performed between the contourlet and the wavelet analysis in terms of result quality and information compaction using a new metric. The potential of the contourlet transform for image compression and further coefficient coding and improvement of its implementation are discussed.

1. INTRODUCTION

During the past two decades, image compression has developed from a mostly academic Rate-Distortion field [18], into a highly commercial business. Various lossless and lossy image coding techniques have been developed [16]. Since the compression ratio obtainable from lossy compression can significantly exceed that obtainable from lossless compression, the primary trade-off concerns the need for reproducibility versus the storage and transmission requirements.

Lossy compression mainly consists of decorrelation and quantization stages that reduce the image size by permanently eliminating certain information. The decorrelation stage of the image compression algorithm is usually done by a transformation from one space to another space to facilitate compaction of information. One approach is the use of multiresolution transforms, that are free from blocking effect artifacts such as in case of the **D**iscrete **C**osine **T**ransform (DCT), which is used in the JPEG (baseline) industry standard [16]. By the use of the **W**a**V**elet **T**ransform (WVT), the corresponding coefficients of the different decomposition levels are correlated and show a characteristic trend. This residual correlation is indicative for a further compression potential. Some standard methods (e.g. JPEG2000) get profits from this potential, especially when considering sets of transform coefficients as feature specific compounds.

Wavelet-based methods have expanded in the field of still image and video compression [16]; they offer the advantage of a better trade-off between complexity, compression and quality over the traditional DCT-based methods. However, for image compression, WVT has a problem with the orientation selectivity because it provides local frequency representation of image regions over a range of spatial scales, and therefore, it does not represent two-dimensional singularities effectively. In a map of the large wavelet coefficients, one sees the edges of the images repeated at scale after scale. This effect means that many wavelet coefficients are required to reconstruct the edges in an image properly, reducing the number of coefficients will introduce artifacts on the edges of the reconstructed image [7].

The **R**id**G**elet **T**ransform (RGT) [5] was developed over several years in an attempt to break an inherent limit plaguing wavelet denoising of images. This limit arises from the frequently depicted fact that the two-dimensional (2-D) WVT of images exhibits large wavelet coefficients to represent the image edges. A basic model for calculating ridgelet coefficients is to view ridgelet analysis as a form of wavelet analysis in the Radon domain. It has been shown in [8] that ridgelet representation solve the problem of sparse approximation of smooth objects with straight edges. In [7], an attempt has been made to use RGT for image compression.

However, in image processing, edges are typically curved rather than straight and ridgelets alone cannot yield efficient representation. But, if one uses a sufficient fine scale to capture curved edges, such an edge gets almost straight, therefore ridgelets are deployed in a localized manner. As a consequence the **C**ur**V**elet **T**ransform (CVT) [4] has been introduced. CVT is based on multiscale ridgelets combined with a spatial bandpass filtering operation. CVT was initially developed in the continuous-domain via multiscale filtering followed by a block RGT on each bandpass image. Later, the authors proposed the second-generation CVT [6] that was defined directly via frequency partitioning without using RGT. Both curvelet constructions require a rotation operation for the frequency decomposition, which ensures the construction in the continuous-domain. For discrete images, sampled on a rectangular grid, the discrete implementation of the curvelet transform is very challenging.

Therefore a new image representation method was introduced: the **C**on**T**ourlet **T**ransform (CTT) [9]. The authors start with a description of the transform in the discrete-domain and then prove its convergence to an expansion in the continuous-domain. Thus a discrete-domain multiresolution and multidirectional expansion is constructed. This in the same way as wavelets are derived from filter banks, but using non-separable filter banks. Due to the fast-iterated filter bank algorithm the construction results in a flexible multiresolution, local and directional image expansion using contour segments. However, CTT has the adverse property of showing other types of artifacts due to the discrete approach.

This paper investigates an assessment of the CTT for image compression, by a combination between CTT and WVT. Due to the evolving development of acquisition technology image size grows up to the order of several million pixels (at least 1 mega-pixel). In order to achieve improved image compression performance a sparse representation is highly required, which considers anisotropy in the image. CTT can be applied efficiently to capture smooth contours at larger resolutions ($\geq$ 256x256 pixels-resolution), while WVT can be used for lower resolution images for further information compaction.

The image quality is evaluated using this combined approach for decorrelation compared to that with wavelets. The potential of CTT for information compaction for high resolution is demonstrated. In Section 2 of this paper, CTT is summarized. Section 3 contains a comparative study between WVT and the CTT-based applied technique on 3 selected high resolution images by means of a quantitative evaluation. Finally, the obtained results are discussed and future directions for image compression are proposed.

2. THE CONTOURLET TRANSFORM

Efficient representations of signals require that coefficients of functions, which represent the regions of interest, are sparse. Wavelets can pick up discontinuities of one dimensional piecewise smooth functions very efficiently and represent them as point discontinuities. 2D WVT obtained by a tensor product of one-dimensional wavelets are good to isolate discontinuities at edge points, but cannot recognize smoothness along contours. Numerous methods were developed to overcome this by adaptive [14], Radon-based [4], or filter bank-based techniques [11]. Do and Vetterli [11] proposed the **P**yramidal **D**irectional **F**ilter-**B**ank (PDFB), which overcomes the block-based approach of CVT by a directional filter bank, applied on the whole scale, also known as CTT. It has been developed to offer the directionality and anisotropy to image representation that are not provided by separable WVT. CTT is a multiscale and directional decomposition of a signal using a combination of a modified **L**aplacian **P**yramid (LP) [3, 10] and a **D**irectional **F**ilter **B**ank (DFB) [12]. PDFB allows for different number of directions at each scale/resolution to nearly achieve critical sampling. As DFB is designed to capture high frequency components (representing directionality), the LP part of the PDFB permits subband decomposition to avoid ”leaking” of low frequencies into several directional subbands, thus directional information can be captured efficiently.

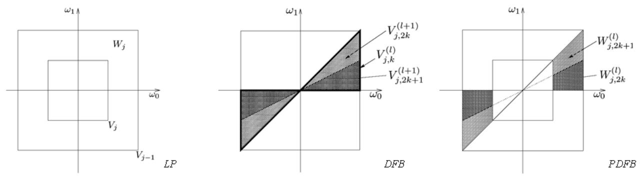

Fig. 1. Generation of subspaces by the LP (left), DFB (middle) and PDFB (right); *j* subspace index; *k* direction; *l* all directions

Fig. 1 illustrates the subspace splitting by respectively LP, DFB and PDFB. V_j is a subspace, defined on a uniform grid with intervals 2^j x 2^j. The difference image in the LP carries the details necessary to increase the resolution from V_j to V_{j-1} on an image approximation; index k runs to all 2^l directions. As stated above, CTT offers the ability to choose the number of directions independently.

We did early experiments with over 100 images, and it was proved, that the smoothness of the contours within an image is coupled with the spatial resolution of a desired scale. We found, that beyond a spatial resolution of 2^8 pixels the application of the CTT carries no advantage compared with WVT in terms of information compaction. Thus, WVT has been used, in this work, instead the CTT for low resolution decomposition levels (scales). Therefore, the adopted image decomposition for compression purpose is then:

1. Four decomposition levels for image size $\leq$ 1024 x 1024 pixels, where two scales are CTT, (with l=16 directions) and the remaining two are WVT (l=3). (See Fig. 2).

2. Five decomposition levels for image size $\leq$ 2048 x 2048 pixels, thus, there are three CTT levels (with l=16 directions) and two WVT levels (l=3);

3. six decomposition levels for image size $\leq$ 4096 x 4096 pixels, thus, there are four CTT levels (with l=16 directions) and two WVT levels (l=3).

In this work, images with size described from item 1. and 2. have been chosen for this investigation and the test results are reported in the next section.

3. EXPERIMENTAL RESULTS

Compression results using WVT and CTT on 3 selected images (”Man”, ”Berry” and ”Art”) are reported in this Section. The evaluation between these approaches is performed using the criterion PSNR and **P**otential **I**nformation **L**oss (PIL) [1].

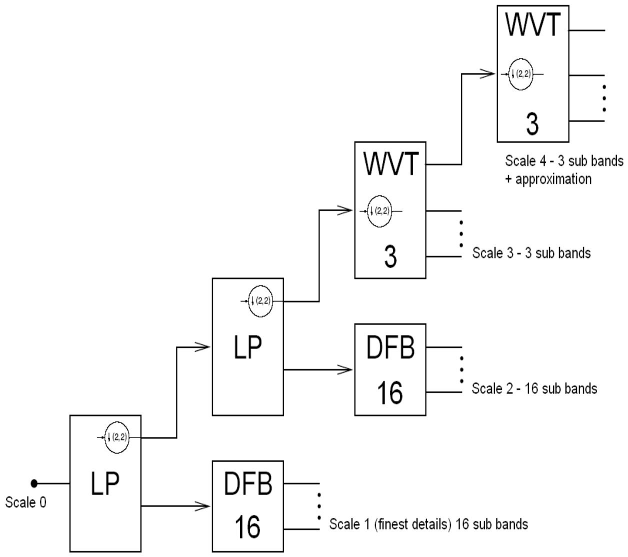

Fig. 2. CTT with four decomposition levels, WVT is used for the two coarse levels.

PIL is a new metric, similar to the Kullback-Leibler distance [1], served for the evaluation of quality of the reconstructed image. It uses the probability density function of the image for evaluation. The histograms in Fig. 3 to Fig. 5 of the reconstructed and those of the original images are used for the calculation of PIL by means of the difference between both histograms (i.e. by computing the sum of the relative differences between all graylevel counts).

The original images are decomposed using CTT (see item 1. and 2. in Section 2) and WVT. Then, simple hard thresholding is performed over delcared insignificant coefficients (with amplitude $<$ a threshold). The images are reconstructed from the remaining significant coefficients and the reconstruction errors are derived. An arithmetic coder [20] is used then to encode the remaining coefficients and the factor between the coefficient sizes from the original and the encoded images are used to derive the compression ratios. Experimental results are presented in Table 1. The number of maintained significant coefficients using CTT is up to 20% smaller; the image quality gets better up to $1dB$ (according to PSNR) than for WVT. Thus, the resulting compression ratio is slightly increased using CTT for an acceptable visual image quality.

Fig. 6 depicts the original selected high resolution images "Man", "Berry" and "Art". Fig. 7, 8 and 9 present the resulting reconstructed images. All *(a)* labeled sub-figures are the WVT results. The WVT artifacts (image quality) can be seen in zoomed regions at the *(b)* labeled sub-figures. The *(c)* labeled sub-figures concern to the CTT results. The CTT image quality can be verified in zoomed regions *(d)*. Additionally to the improved information compaction by CTT, visual quality preservation (better representation of edges) can be seen from the zoomed images (d) compared to WVT results (b). A reason for that can be noticed from Fig. 4, such that CTT

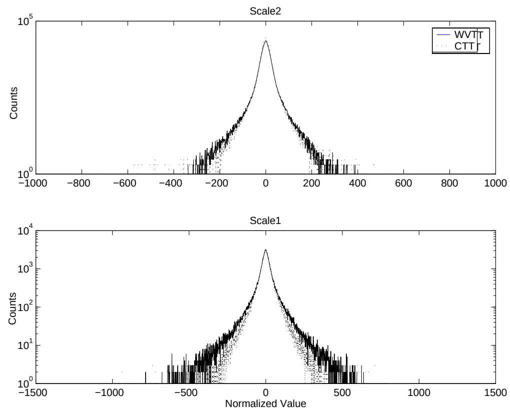

Fig. 3. The histograms of the coefficients "Man" image.

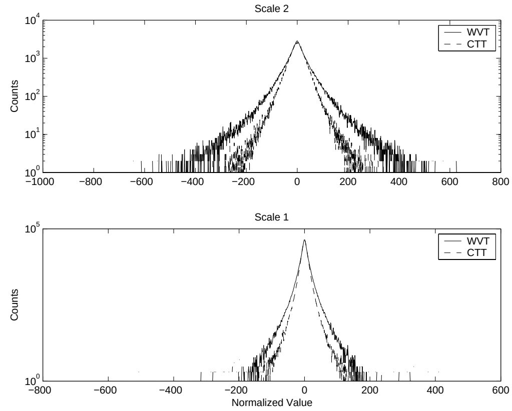

Fig. 4. The histograms of the coefficients "Berry" image.

provides a sparser representation of the images at fine scales and therefore, compacts them in fewer coefficients. At coarse scales, usually for $\leq$ 256 x 256 pixel-images, WVT can outperform CTT as object geometry is simplified and therefore, contours are not smooth anymore.

4. CONCLUSION AND PERSPECTIVES

This paper investigates the potential of the **C**on**T**ourlet **T**ransform (CTT) for compression of high resolution images ($\geq$ 1 million pixels). The experimental results show a promising perspective as CTT provides a more compact representation of the information compared to that of the **W**a**V**elet **T**ransform (WVT) used in the compression standard JPEG2000. This is figured out by the histograms of the coefficients at Fig. 3 to Fig. 5, which mostly shows the CTT inside the WVT curves. Furthermore, CTT shows less information loss and artifacts on the reconstructed images after a simple hard thresholding of the transformed coefficients.

Indeed, WVT exhibits a large number of coefficients for representing smooth contours, which are usually present in high resolution images. However, WVT compacts better the

Images	Image Size	Nb, of Maintained Coefficients	PSNR dB	PIL	Compression Ratio
”Man” by WVT	1k x 1k	48522	24.5	0.62	2.8
”Man” by CTT	1k x 1k	43242	25.5	0.42	3.2
”Berry” by WVT	1k x 1k	5869	25.9	0.24	28.1
”Berry” by CTT	1k x 1k	4668	26.4	0.20	29.3
”Art” by WVT	2k x 2k	16855	23.0	0.3	31.4
”Art” by CTT	2k x 2k	15248	24.1	0.3	33.1

Table 1. Number of maintained coefficients, PSNR, PIL and CR for the test images.

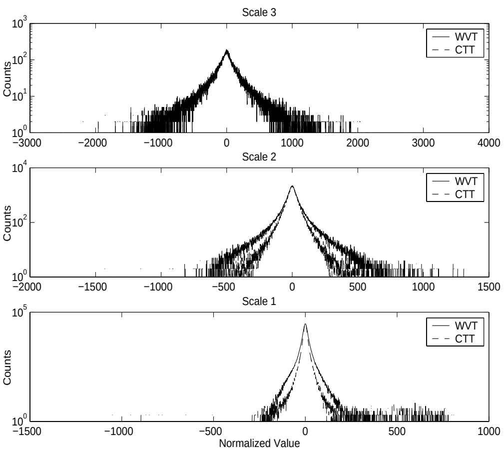

Fig. 5. The histograms of the coefficients ”Art” image.

information for the coarse resolution (up to 2^8 pixel-image) as the geometry, usually, gets simplified. Therefore, a combination CTT-WVT (CTT for finest resolution and WVT for coarse resolution) seems to be good candidate for a new compression codec. Also a metric should be developed to find the objective best scale for the turnover from CTT to WVT processing. Investigations are still needed to recognize the adequate entropy coder for the resulting coefficients. Indeed, the hierarchical structure exploited by the JPEG 2000 codec is not explicitly valid for our combination of methods.

In a future work the combination of the coefficients from both transforms into zero trees (representing the interscale dependencies) will be studied.

5. REFERENCES

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a

b

c

Fig. 6. The original images.

a

b

c

d

Fig. 7. Reconstructed ”Man” images (at right).

a

b

c

d

Fig. 8. Reconstructed ”Berry” images.

a

b

c

d

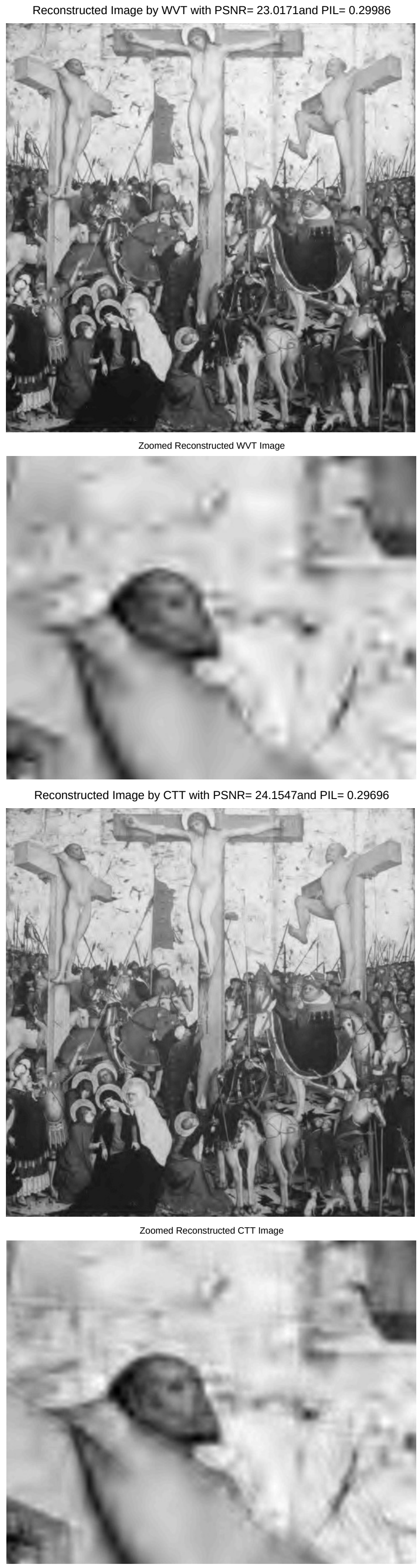

Fig. 9. Reconstructed ”Art” images.