EVALUATION OF THE ITERATIVE LEAST SQUARE METHOD IN DIGITAL FILTER DESIGN

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ABSTRACT

In this paper we outline the experiments of the Iterative Least Square Direct algorithm on several filters. The experiments of these methods, compared to standard method such as Tabu Search method, demonstrate their feasibility and good performance. It is also shown the low algorithmic complexity of this novel approach.

I. INTRODUCTION

Many large-volume electronic consumer products are based on digital signal processing (DSP). Digital filters are integral parts of many digital processing systems, including communication systems, control systems, systems for audio and image processing and systems for medical applications. In DSP systems, the signal that contains the information of interest is represented by a sequence of numbers, so called samples.

The DSP system operates on this input sequence of numbers to form an output sequence. In general, the overall aim of a signal processing system is to reduce the information content, or to modify it so that it can be efficiently stored or transmitted over a transmission channel.

Examples of such DSP algorithms are digital filters, fast Fourier transform (FFT), discrete cosine transformation (DCT), and wavelet transforms.

In this paper, we are mainly interested on the first case, and especially on the filter design directly on the discrete space. Many standard methods for this purpose use the Chebyshev approximation. As an example, Remez Exchange Algorithm is usually applied for the design of infinite precision linear phase (FIR) filters [9]. For (DSP) implementation, the most widely used approach to the problem is the rounding of the optimal infinite precision coefficients to its wordlength representation. However, the filters obtained are degraded and do not fulfill the spectral requirements for which they are expected.

Sequential and Progressive Search method (SPS) [13] is performed for the design of finite precision (FIR) filters for both minmax and least square approximation. Although it is possible to obtain optimal filter, the algorithm is unfeasible for long filter order.

The Tabu Search (TS) algorithm [3],[4],[5],[6] has proven high performance and low algorithmic complexity. Especially for the latter case, it seems that no algorithm could cope with (TS) used in the optimisation problem. The (TS) tool has also proven to be both versatile and easy to use, thus rapidly allowing its customisation to different optimisation applications. It exploits some of the most effective search techniques taken from the literature, as well as some new search strategies. In the case of digital FIR filter design, all (TS) algorithm applications have been used on the minmax sense. This paper is formulated with convenience that versatile methods performances are not affected by the approximation criterion. Least square approximation is the criterion of the choice of much DSP application where minimising error energy or error power is desired.

In this paper, we present digital filter design method based on least square approximation. It is called Iterative Least Square Direct method (ILSD) [1]. The method purpose is the filter design solution updating using several iterations. We have chosen the Tabu Search (TS) algorithm in order to compare the (ILSD) algorithm performance and cost function.

In section II, we present the problem statements and the characteristics of the error criterion chosen. In section III, an overview of the (ILSD) method is given. The algorithm modules are depicted in section IV. The results reported on section V deal with conventional Least Square optimisation of FIR digital filters and are compared to those of Tabu Search method.

II. PROBLEM STATEMENTS

Let us consider the design of N-1 order linear phase FIR digital filter with a frequency response H(f) usually written as

$$H(f) = \sum_{k=0}^{N-1} h_k e^{-j2\pi fk} \tag{1}$$

In [8], It was shown that the frequency response amplitude of the four cases of linear phase (FIR) filters could be written in the form

$$P_n(f) = \sum_{k=0}^{n-1} a_k \cos 2\pi fk \tag{2}$$

Where the number of terms, n, is:

n(= N/2 or (N-1)/2 or (N+1)/2)

And a_k is the resulting shifted sequence depending on the considered case. The function $P_n(f)$ is compared with a desired frequency response amplitude D(f) using the least square criterion. The approximation error e_n is given by

$$e_n = \frac{1}{Nf}\sum_{i=0}^{Nf} \left|D(f_i) - P_n(f_i)\right|^2 \qquad (3)$$

- i = 1,2,…, Nf-1
- Nf : Number on Sample frequency selected.
- D(f) : the desired frequency response amplitude. For an ideal low pass filter we have

$D(f_i) = 1$ if $f_i \in$ bandpass.
$D(f_i) = 0$ if $f_i \in$ stopband .

Hence, Eq. (3) will be

$$e_n = \frac{1}{Nf}\left\{ \sum_{i=0}^{k-1}\left\|1 - P_n(f_i)\right\|^2 + \sum_{i=k}^{Nf-1}\left|P_n(f_i)\right|^2 \right\} \qquad (4)$$

The filter coefficients are restricted to the discrete values allowed by (bwl) bit binary word length. In this paper we have chosen to use the fixed-point and power of two representations. The power of two representation yields a set of admissible values (D) which is defined as follow :

$$D = \left\{ \begin{array}{l} \alpha : \alpha = \sum_{k=1}^{2} c_k \cdot 2^{-g_k}, \quad c_k \in \{-1,0,1\}, \\ g_k \in \{1,2,\ldots,b\} \end{array} \right\} \qquad (2)$$

where b is the maximum number of shifts.

III. OPTIMIZATION METHOD 'ITERATIVE LEAST SQUARE DIRECT METHOD' (ILSD)

The (ILSD) method [1] copes with two main problems in the discrete filter design:

- The Least Square method (LSD) [2], [14] problem, which consists on the choice of the starting point and the order in which the other coefficients are considered.
- The problem of Depth First Search [14], which is the prohibitive computing time in the case of long filter order.

Therefore, this method performs in many iterations in order to improve the (LSD) performances.
To describe the (ILSD) method, we choose the following example:

Let us consider a filter defined by two coefficients {h(0), h(1)}. The processor wordlength provided is (bwl) and 'av' admissible values could be represented.
The (ILSD) method runs as follows:
First, (LSD) is used to calculate the coefficients $h_{lsd}(0)$ and $h_{lsd}(1)$. Let us denote the filter design error by E_r.
$h_{lsd}(0)$ is fixed, then $h_{lsd}(1)$ will vary in d-diameter discrete values range centred around $h_{lsd}(1)$ (d<av).
Indeed, at each discrete combining, the least square design error is calculated. Let us denote it by E_{ilsd}.
If $E_{ilsd} < E_r$, then $E_r = E_{ilsd}$ and will be the reference error.
At the same manner, $h_{lsd}(1)$ is now fixed and $h_{lsd}(0)$ will vary in d discrete values range. E_{ilsd} is calculated again and compared to E_r.
This procedure is iterated many times till the error variation will vanish. Let ΔEr is the error variation:

$$\Delta Er = Er(\text{ actual iteration}) - Er(\text{previous iteration}) \qquad (5)$$

Which is provided by the final filter coefficients ($h_{ilsd}(0)$, $h_{ilsd}(1)$) after (It) iterations. Figure 1 represents ILSD method after two iterations.
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IV. THE ALGORITHM

The (ILSD) algorithm is subdivided in two parts as depicted in Figure 2:

- The first part represents the (LSD) algorithm outlined in [2], [14], in which the filter coefficients are sequentially computed.

- The second part represents the iterative method which emphases are the (LSD) performance improvement. This is due by a local search of the new solution providing a low error design in the neighbourhood of the previous. Regarding the wordlength, the parameter (d) is fixed.

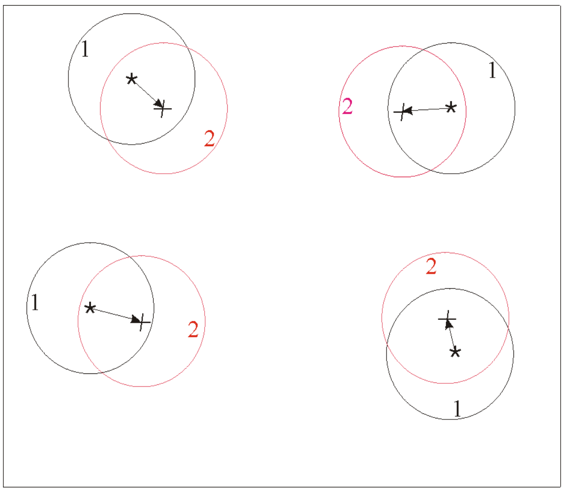

*Figure 1. ILSD Procedure on four coefficients in the Discrete Space (1: First Iteration, 2:Second Iteration, *:LSD solution, +: ILSD solution after one iteration)*

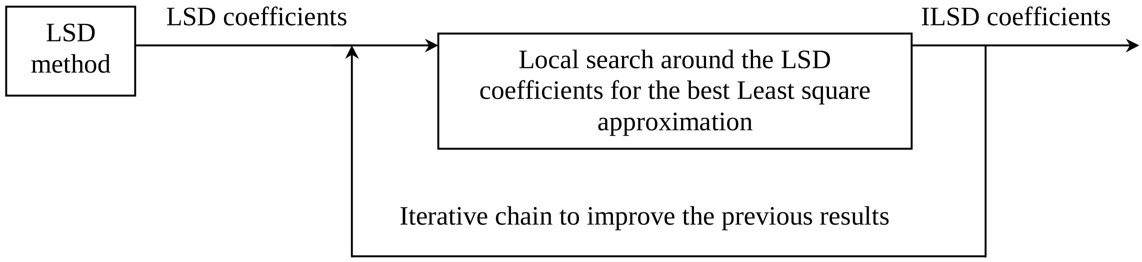

Figure 2. The procedure of the Iterative Least Square Direct method

Filter length/Wordlength	Infinite precision	Rounded	TS/ Time (sec)	ILSD/ Time (sec)
8/7	0.0374	0.0358	0.0358/ 2.02	0.0356/ 1.9
8/15	0.0374	0.0374	0.0372/ 30.22	0.0314/ 11
8/19	0.0374	0.0374	0.0373/ 14.25	0.0314/ 302
16/15	0.0706	0.0706	0.0596/ 73.47	0.0389/ 120
16/15	0.0829	0.0829	0.0818/ 78.24	0.0472/ 163
56/13	0.0049	0.0049	0.0049/ 465	0.0030/ 153

Table 1. Results of ILSD Filter Design compared to TS in fixed-point representation.

Filter length/Wordlength	Infinite precision	Rounded	TS/ Time (sec)	ILSD/ Time (sec)
8/7	0.0374	0.1270	0.1270/ 10.84	0.1270/ 0.88
8/15	0.0374	0.0624	0.0556/ 10.06	0.0512/ 55
8/19	0.0374	0.0624	0.0477/ 10.37	0.0461/ 110
16/15	0.0706	0.0748	0.0748/ 47.35	0.0399/ 80
16/15	0.0829	0.0847	0.0704/ 71.57	0.0524/ 62
56/13	0.0049	0.0236	0.0236/ 415	0.0173/ 85

Table 2. Results of ILSD Filter Design compared to TS in power of two representation.

V. RESULTS

The (ILSD) algorithm has been tested and compared to Tabu Search algorithm using cases reported in literature. The software algorithm was developed in MATLAB and tested on a 300 MHz Pentium machine. A filter with length 8 with 7 bits in quantization, excluded the sign bit is denoted by '8/7'. The starting points for (TS) algorithm are chosen depending on the filter case. They could be Parks-Mc Clellan coefficients, random values or zero-values. Table 1 and Table two represent the application of (ILSD) using the fixed point and power of two representations respectively. The three first filters in the Table 1 and 2 have the passband edges (0,0.159) and the stopband edges (0.259,0.5). The fourth filter has the passband edges (0,0.318) and the stopband edges (0.371,0.5), the fifth filter has (0,0.05) passband edges and (0.104,0.5) stopband edges and the last filter has the passband edges (0,0.31) and the stopband edges (0.35,0.5). All the filters have equal weights in passbands and stopbands. In all examples the (ILSD) performances are better than those obtained with Tabu Search. The computing time varies from a example to another. The ILSD algorithm presents low algorithmic complexity than (TS) algorithm when long filter order is used. In the case of large processor wordlength, the (ILSD) computing time is relatively longer than that of (TS) but still remain reasonable (hundred seconds). In general, using both factors performance and computing time, it is clear that (ILSD) takes the advantage.

VI. CONCLUSION

In this paper, an evaluation of the Iterative Least Square Direct algorithm (ILSD) for digital filter design on several examples is presented. The concept is mainly based on an iterative filter approximation in the coefficients discrete space. The comparison with Tabu Search (TS) algorithm demonstrates the good (ILSD) algorithm performance and large applicability to a high range of filters. Furthermore, (ILSD) presents lowest algorithmic complexity. The next forthcoming step consists on how to well define the local discrete diameter in order to guarantee the optimality. A deeper mathematical study is on going.

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